Practice Makes Imperfect
How Problem Distribution Can Lead to Prototype Formation

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STUDY OF CHILDREN'S THINKING

Introduction

- Math learners have difficulty generalizing to novel problems despite substantial practice
- Mental models of problem-solving that learners construct reflect sensitivity to predictive relations of practice problems
- Patterns of frequency distributions in practice can cause learners to form prototype-like representations of math problems, leading to inappropriate generalization
- Examples of prototype formation in math
  - 8 is "more even" than 1132 (Armstrong et al. 1983)
  - Equilateral triangles are "best" triangles (Knuth et al. 2012)
- Practice experiences that highlight quantitative relations may protect learners against forming prototype representations and making generalization errors

Hypotheses

1. **Symbolic practice will lead to prototype formation, resulting in more errors on low-frequency and lure problems**
2. **Grounded practice will insulate learners from forming prototypes, leading to fewer errors on all problems, including lures**
3. **Grounded practice will yield richer representation of problem structure, leading to better transfer on problems with novel quantities**

Method

Participants
60 adult undergraduates at UW-Madison

Procedure
Participants played a computer-based math game using base-8 alphabetic addition, and were randomly assigned to a condition that either emphasized quantitative relationships between the symbols *(Grounded)* or a symbol-only problem structure *(Symbolic)*.

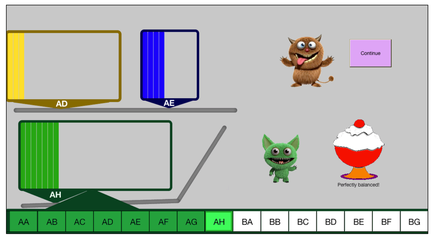
Training Phase
Participants were trained on practice problems with unequal frequency distributions.

- High-frequency problems: repeated 20x
- Low-frequency problems: repeated 5x

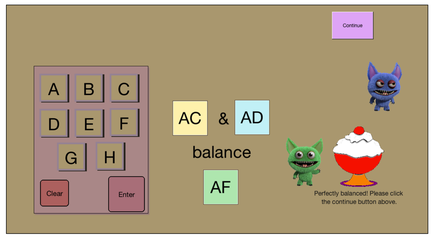
Testing Phase
- Familiar problems (High- & Low-frequency)
- Lure problems: contained addend from high-frequency problems
- Novel problems

Monster Ice Cream Game

Grounded Training

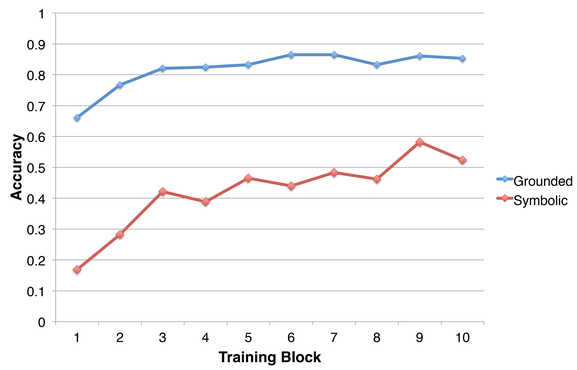


Symbolic Training

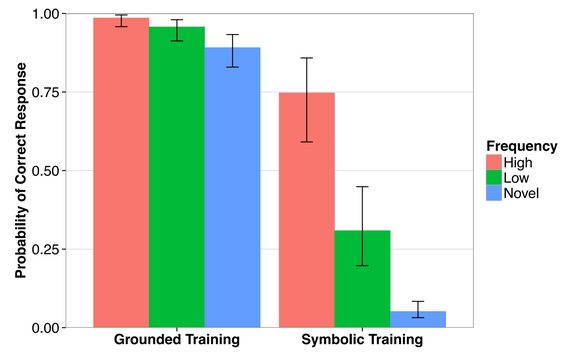


Results-Adults

Training Phase

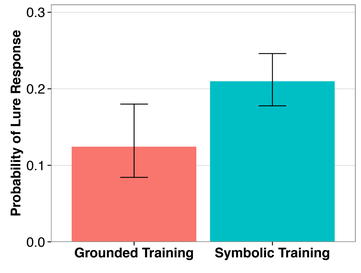


Test Phase



- Frequency effect for Symbolic training only
- Grounded training more accurate overall

Lure problems

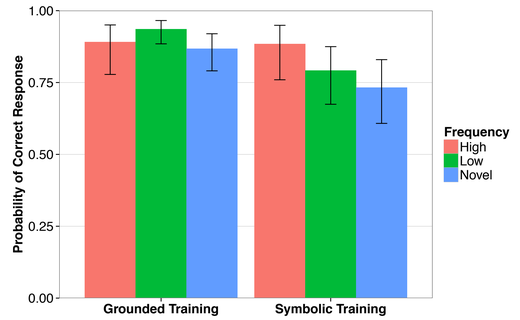


- Symbolic Training: ~2x as likely to make a lure response

Results-Children

Pilot Data
- 42 second-grade children
- Practice and test on typical arithmetic problems

Participants in the Grounded practice condition were equally successful on both frequent and novel quantity problems.



Future Directions

- Manipulate problem types (e.g., 19 + 6 = ? → 19 + ? = 25)
- Evaluate format transfer: Does Grounded training help Symbolic problem-solving?
- Train children on more difficult problems to avoid familiarity effects

References

Armstrong, S. L., Gleitman, L. R., & Gleitman, H. (1983). What some concepts might not be. *Cognition*, *13*(3), 263-308.

Knuth, E. J., Kalish, C. W., Ellis, A., Williams, C., & Felton, M. (2012). Adolescent Reasoning in Mathematical and Non-Mathematical Domains: Exploring the Paradox. In V. F. Reyna, S. B. Chapman, M. R. Dougherty & J. Confrey (Eds.), *The Adolescent Brain: Learning, Reasoning, and Decision Making*. Washington, DC: APA.

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